

MATHEMATICS

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**XIth, XIIth, TARGET IIT-JEE
(MAIN + ADVANCE) & COMPETITIVE EXAM.
FOR XII (PQRS)**

DERIVATIVE AS A RATE MEASURER & Their Properties

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EXERCISE-1

1. Find the rate of change of the area of a circle with respect to its radius. How fast is the area changing with respect to the radius when the radius is 3 cm ?
2. The total cost $C(x)$ associated with the production of x units of an item is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$
Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output.
3. The total revenue received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. Find the marginal revenue when $x = 5$, where by marginal revenue we mean the rate of change of total revenue with respect to the number of items sold at an instant.
4. The total cost $C(x)$ associated with the production of x units of an item is given by $C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000$. Find the marginal cost when 17 units are produced.
5. The total revenue received from the sale of x units of a product is given by $R(x) = 13x^2 + 26x + 15$. Find the marginal revenue when $x = 7$.
6. The radius of a circle is increasing uniformly at the rate of 4 cm/sec. Find the rate at which the area of the circle is increasing when the radius is 8 cm.
7. The radius of a balloon is increasing at the rate of 10 cm/sec. At what rate is the surface area of the balloon increasing when the radius is 15 cm ?
8. A stone is dropped into a quiet lake and waves move in a circle at a speed of 3.5 cm/sec. At the instant when the radius of the circular wave is 7.5 cm, how fast is the enclosed area increasing ?
9. A particle moves along the curve, $6y = x^3 + 2$. Find the points on the curve at which the y -coordinate is changing 8 times as fast as the x -coordinate.
10. The volume of a cube is increasing at a rate of 7 cm³/sec. How fast is the surface area increasing when the length of an edge is 12 cm ?
11. The volume of a cube is increasing at a constant rate. Prove that the increase in surface area varies inversely as the length of the edge of the cube.
12. A man 2 metres high, walks at a uniform speed of 6 metres per minute away from a lamp post, 5 metres high. Find the rate at which the length of his shadow increases.
13. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground away from the wall, at the rate of 2 m/sec. How fast its height on the wall decreasing when the foot of the ladder is 4 m away from the wall ?
14. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm/sec. How fast is the area decreasing when the two equal sides are equal to the base ?
15. The length x of a rectangle is decreasing at the rate of 2 cm/sec and the width y is increasing at the rate of 2 cm/sec. When $x = 12$ cm and $y = 5$ cm, find the rate of changes of (i) the perimeter and (ii) the area of the rectangle.

16. Water is dripping out from a conical funnel of semi-vertical angle $\frac{\pi}{4}$ at the uniform rate of $2 \text{ cm}^2/\text{sec}$ in its surface area through a tiny hole at the vertex in the bottom. When the slant height of the water is 4 cm, find the rate of decrease of the slant height of the water.
17. Sand is pouring from a pipe at the rate of $12 \text{ cm}^3/\text{sec}$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand-cone increasing when the height is 4 cm ?
18. A man is walking at the rate of 6.5 km/hr towards the foot of a tower 120 m high. At what rate is he approaching the top of the tower when he is 50 m away from the tower ?
19. An inverted cone has a depth of 10 cm and a base of radius 5 cm. Water is poured into it at the rate of $3/2$ c.c. per minute. Find the rate of which the level of water in the cone is rising when the depth is 4 cm.
20. Water is dripping out from a conical funnel at a uniform rate of $4 \text{ cm}^3/\text{sec}$ through a tiny hole at the vertex in the bottom. When the slant height of the water is 3 cm, find the rate of decrease of the slant height of the water-cone. Given that the vertical angle of the funnel is 120° .
21. Water is running into a conical vessel, 15 cm deep and 5 cm in radius, at the rate of $0.1 \text{ cm}^3/\text{sec}$. When the water is 6 cm deep, find at what rate is
 - (i) the water level rising ?
 - (ii) the water-surface area increasing ?
 - (iii) the wetted surface of the vessel increasing ?
22. A man is moving away from a tower 41.6 m high at the rate of $2 \text{ m}/\text{sec}$. Find the rate at which the angle of elevation of the top of tower is changing, when he is at a distance of 30 m from the foot of the tower. Assume that the eye level of the man is 1.6 m from the ground.
23. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lower most. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic metre per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4 m.
24. The side of a square sheet is increasing at the rate of 4 cm per minute. At what rate is the area increasing when the side is 8 cm long ?
25. The radius of a circle is increasing at the rate of 0.7 cm/sec. What is the rate of increase of its circumference ?
26. A balloon which always remains spherical, is being inflated by pumping in 900 cubic centimetres of gas per second. Find the rate at which the radius of the balloon is increasing when the radius is 15 cm.
27. A stone is dropped into a quiet lake and waves move in circles at a speed of 4 cm/sec. At the instant when the radius of the circular wave is 10 cm, how fast is the enclosed area increasing ?

28. A man 180 cm tall walks at a rate of 2 m/sec. away, from a source of light that is 9 m above the ground. How fast is the length of his shadow increasing when he is 3 m away from the base of light ?
29. A particle moves along the curve $y = x^2 + 2x$. At what point(s) on the curve are the x and y coordinates of the particle changing at the same rate ?
30. The top of a ladder 6 metres long is resting against a vertical wall on a level pavement, when the ladder begins to slide outwards. At the moment when the foot of the ladder is 4 metres from the wall, it is sliding away from the wall at the rate of 0.5 m/sec. How fast is the top-sliding downwards at this instance ? How far is the foot from the wall when it and the top are moving at the same rate ?
31. The surface area of a spherical bubble is increasing at the rate of $2 \text{ cm}^2/\text{s}$. When the radius of the bubble is 6 cm, at what rate is the volume of the bubble increasing ?
32. A particle moves along the curve $y = (2/3)x^3 + 1$. Find the points on the curve at which the y-coordinate is changing twice as fast as the x-coordinate.
33. Find the point on the curve $y^2 = 8x$ for which the abscissa and ordinate change at the same rate.
34. A circular disc of radius 3 cm is being heated. Due to expansion, its radius increases at the rate of 0.05 cm/sec. Find the rate of which its area is increasing when radius is 3.2 cm.
35. A car starts from a point P at time $t = 0$ seconds and stops at a point Q. The distance x, in meters, covered by it, in t seconds is given by $x = t^2 \left(2 - \frac{t}{3} \right)$. Find the time taken by it to reach Q and also find distance PQ.

EXERCISE-2

1. A cylindrical vessel of radius 0.5 m is filled with oil at the rate of $0.25 \pi \text{ m}^3/\text{minute}$. The rate at which the surface of the oil is rising, is
 - (a) 1 m/minute
 - (2) 2 m/minute
 - (c) 5 m/minute
 - (d) 1.25 m/minute
2. The altitude of a cone is 20 cm and its semi-vertical angle is 30° . If the semi-vertical angle is increasing at the rate of 2° per second, then the radius of the base is increasing at the rate of
 - (a) 30 cm/sec
 - (b) $\frac{160}{3}$ cm/sec
 - (c) 10 cm/sec
 - (d) 160 cm/sec
3. The coordinates of the point on the ellipse $16x^2 + 9y^2 = 400$ where the ordinate decreases at the same rate at which the abscissa increases, are
 - (a) $\left(3, \frac{16}{3} \right)$
 - (b) $\left(-3, \frac{16}{3} \right)$
 - (c) $\left(3, -\frac{16}{3} \right)$
 - (d) (3, -3)
4. The radius of a sphere is increasing at the rate of 0.2 cm/sec. The rate at which the volume of the sphere increases when radius is 15 cm, is
 - (a) $12\pi \text{ cm}^3/\text{min}$
 - (b) $7\pi \text{ cm}^3/\text{min}$
 - (c) $27\pi \text{ cm}^3/\text{min}$
 - (d) none of these

5. The distance moved by a particle travelling in a straight line in t second is given by $s = 45t + 11t^2 - t^3$. The time taken by the particle to come to rest is
(a) 9 sec (b) $5/3$ sec (c) $3/5$ sec (d) 2 sec
6. The rate of change of volume of a sphere is equal to the rate of change of its radius, then its radius is equal to
(a) 1 unit (b) $\sqrt{2\pi}$ units (c) $\frac{1}{\sqrt{2\pi}}$ unit (d) $\frac{1}{2\sqrt{2\pi}}$ unit
7. If $s = t^3 - 4t^2 + 5$ describes the motion of a particle, then its velocity when the acceleration vanishes, is
(a) $\frac{16}{9}$ unit/sec (b) $-\frac{32}{3}$ unit/sec (c) $\frac{4}{3}$ unit/sec (d) $-\frac{16}{3}$ unit/sec
8. The equation of motion of a particle is $s = 2t^2 + \sin 2t$, where s is in metres and t is in seconds. The velocity of the particle when its acceleration is 2 m/sec^2 , is
(a) $\pi + \sqrt{3}$ m/sec (b) $\frac{\pi}{3} + \sqrt{3}$ m/sec (c) $\frac{2\pi}{3} + \sqrt{3}$ m/sec (d) $\frac{\pi}{3} + \frac{1}{\sqrt{3}}$ m/sec